

# AdS Holography and Strings on the Horizon

by

**A.J.M. Medved**

*Department of Physics and Theoretical Physics Institute  
University of Alberta  
Edmonton, Canada T6G-2J1  
[e-mail: [amedved@phys.ualberta.ca](mailto:amedved@phys.ualberta.ca)]*

## ABSTRACT

The microscopic origin of black hole entropy remains one of the more intriguing open questions in theoretical physics. A subplot in this drama is the renowned Cardy-Verlinde formula, which uses two-dimensional conformal formalism to explain the entropy of an arbitrary-dimensional black hole. In this paper, by exploiting the AdS/CFT and black hole-string dualities, we are able to provide a physical picture for this paradoxical behavior. Following a recent study by Haylo (in a dS context), we show that the dual CFT for an asymptotically AdS spacetime actually conforms to a string-like description. Moreover, we demonstrate that this stringy CFT is directly related to a string that lives on the stretched horizon of an AdS-Schwarzschild-like black hole. In fact, after an appropriate renormalization, these two boundary theories are shown to be thermodynamically equivalent.

# 1 Introduction

One of the major tests for any prospective theory of quantum gravity will be its ability to explain the Bekenstein-Hawking (black hole) entropy [1, 2] from a microscopic perspective. That is to say, the definitive statistical-mechanical explanation for this entropy, which has its origins in purely thermodynamic principles, is still conspicuously lacking. (See Ref.[3] for a general overview on this puzzlement and recent attempts at its resolution.)

Many microscopic formulations of black hole entropy have utilized the well-known Cardy formula [4] as a key, intermediate step. In fact, some recent treatments by Carlip [5] and Solodukhin [6] have shown that the Cardy formula is relevant to the entropic calculation of any theory that admits a black hole horizon. (Although there may be some yet-unknown pathological exceptions.) What is both intriguing and, at the same time, puzzling is that, strictly speaking, the Cardy formula counts the degrees of freedom in a two-dimensional conformal field theory (CFT). Paradoxically, the cited methods apply to virtually any black hole of arbitrary dimensionality. That is to say, the black hole horizon should, naively, only be expected to host a two-dimensional effective theory in the special case of a three-dimensional spacetime. This logic follows from the holographic principle [7, 8], which implies that a  $d+1$ -dimensional bulk theory can effectively be described by a  $d$ -dimensional boundary theory.<sup>1</sup>

Interestingly, the Cardy formula plays a significant role in yet another facet of black hole entropy. Not too long ago, Verlinde [10], exploiting the well-known duality between anti-de Sitter (AdS) spacetimes and CFT's [11, 12, 13], was able to show that the entropy of an AdS-Schwarzschild black hole can be expressed in a Cardy-like form. That is, given such a  $d+1$ -dimensional asymptotically AdS spacetime, the thermodynamic properties of an appropriately dual ( $d$ -dimensional) CFT can be arranged into a replica of the Cardy formula; provided that a suitable identification with the Cardy “central charge” [4] has been made. It just so happens that this Cardy-like entropy of the CFT is identical to that of the AdS-black hole bulk [13]. (Note that the dual nature of such a CFT, which lives on a  $d$ -dimensional timelike boundary of the AdS bulk, follows from the AdS/CFT correspondence [11,

---

<sup>1</sup>Alternatively, this logic follows from a commonly accepted viewpoint: the degrees of freedom of a black hole will predominantly live at or near the horizon [9].

12, 13]. This correspondence, in turn, can be viewed as a direct consequence of the holographic principle [7, 8].)

Verlinde’s particular formulation [10] of the AdS/CFT entropy is now commonly referred to as the Cardy-Verlinde formula. This basic outcome has since been generalized for a plethora of asymptotically AdS theories (for instance, [14]), including dynamical-boundary scenarios (for instance, [15]).<sup>2</sup> Furthermore, the Cardy-Verlinde formula has even been extended to asymptotically de Sitter (dS) spacetimes (for instance, [17]<sup>3</sup>), although with only qualified success.<sup>4</sup>

Just like the microscopic entropy calculations of (for instance) Carlip and Solodukhin [5, 6], the success of the Cardy-Verlinde formula (and its myriad of generalizations) leads to a puzzling issue. Namely, why can the degrees of freedom for a  $d$ -dimensional field theory (albeit a conformal one) be consistently explained in terms of two-dimensional conformal formalism? This will undoubtedly be a difficult question to answer, given the current absence of any reliable theory of quantum gravity. Very recently, however, Haylo did manage to shed some light on the matter [20]. In the context of a  $d+1$ -dimensional asymptotically dS bulk, this author demonstrated that a dual ( $d$ -dimensional) CFT exhibits string-like behavior. In particular, the CFT (or dS-bulk) entropy was shown to be linearly related to the extensive energy of the boundary theory. On the basis of this result, Haylo identified an effective string tension for the CFT. With this stringy description, it follows that the  $d$ -dimensional boundary theory can alternatively be viewed as having only two relevant dimensions.

Haylo went on to show [20] that the CFT thermodynamic properties (including the effective string tension) are directly related to those of a string that lives on the “stretched” (cosmological) horizon<sup>5</sup> of the asymptotically dS bulk. (This latter picture follows from a black hole-string correspondence

---

<sup>2</sup>For a thorough list of many of the relevant citations, see Ref.[16].

<sup>3</sup>For further citations along this line, consult Ref.[18].

<sup>4</sup>In this regard, it is worth noting that the (proposed) dS/CFT correspondence [19] remains on somewhat weaker ground than its AdS analogue. This can especially be attributed to the lack of a globally timelike Killing vector and the lack of a string-theoretical description. For further references on the dS/CFT duality, see Ref.[18].

<sup>5</sup>A stretched black hole (or cosmological) horizon [21, 22] essentially refers to a horizon *per se*, along with any point separated by a spacelike distance less than the fundamental string length ( $l_s$ ).

that was originally proposed by Susskind [21].) Moreover, when the thermodynamics at the stretched horizon have properly been renormalized (to account for the large gravitational red shift occurring between the horizon and the CFT boundary), then the two stringy descriptions turn out to be equivalent. That is, the CFT can, at least effectively, be interpreted as a horizon-based string with a renormalized tension. Within this inherently two-dimensional framework, a Cardy-like formulation of the CFT entropy begins to make sense.

Before proceeding to an outline of the current paper, a brief discussion on the black hole-string correspondence is in order. Roughly a decade ago, Susskind conjectured [21] a one-to-one correspondence between sufficiently massive black holes and highly excited fundamental strings. This notion was based on the observation [23] that, as string coupling increases, the size of a string state must eventually become less than its Schwarzschild radius and, hence, the string state must evolve into a black hole. Conversely, as string coupling decreases, the size of a black hole will fall below the string scale and, hence, the near-horizon geometry can no longer be interpreted as a black hole. It follows that a black hole and some sort of string configuration must be strongly and weakly coupled manifestations of the same entity. Now consider that, at large enough mass, a typical string state consists of a small number of highly excited strings. However, as such a configuration approaches the high-temperature conditions of the horizon, the state of a single string will become entropically preferred. Alternatively, an external observer might say that the strings have melted together into a single string that ultimately fills up the stretched horizon [24]. Given these arguments, there should indeed be a one-to-one correspondence between black holes and strings.

Contrary to the above philosophy, there is, of course, an observed discrepancy between black hole and string thermodynamics. In particular, the entropy of a string has a linear energy dependence while, for instance, the entropy of a four-dimensional Schwarzschild black hole has a quadratic dependence. However, Susskind proposed [21] that this discrepancy can be accounted for when the energy of the string has properly been renormalized. Such a renormalization follows naturally by virtue of a large gravitational red shift that occurs between the stretched horizon and an external observer. Exploiting the Rindler-like near-horizon geometry (of a Schwarzschild black hole), Susskind went on to demonstrate the feasibility of this proposal.

The validity of this black hole-string correspondence has since been rigor-

ously substantiated for a wide range of black hole (as well as brane) scenarios [25, 26, 27, 28, 29, 30, 31]. Generally speaking, the entropy obtained from the counting of string states has been shown to describe the anticipated Bekenstein-Hawking form up to some ambiguity in the numerical coefficient. It is worth noting that the same philosophy has been applied to special classes of supersymmetric extremal and near-extremal black holes. (See, for instance, Ref.[32] and, for a review, Ref.[33].) For these calculations, the Bekenstein-Hawking form has been reproduced exactly. However, a special property of such models (namely, supersymmetry protects the mass against renormalization [34]) prevents a more general application of these particular methods.

Now that the necessary background has (hopefully) been covered, let us focus on the purpose of the current study. Here, we will essentially be extending Haylo's priorly discussed treatment [20] to the case of an asymptotically AdS bulk (of arbitrary dimensionality). That is, we will demonstrate that an appropriately dual CFT can effectively be described by a string that lives on the stretched black hole horizon. Although this is, perhaps, a trivial extension, we believe that it is important to clarify the success (or lack thereof) of this procedure for an AdS-bulk scenario. Our rationale being as follows. If any progress is to be made in extrapolating this outcome towards a microscopic realization of black hole entropy, then it will likely come in an AdS (rather than dS) context, where the holographic duality is much better understood.

The remainder of this paper is organized as follows. In Section 2, we begin by introducing the bulk solutions of interest; namely, arbitrary-dimensional AdS-Schwarzschild black holes and their topological variants [35]. Next, we derive the relevant black hole thermodynamics and, following Verlinde [10], obtain the corresponding expressions for a dually related CFT. We then demonstrate the anticipated string-like behavior of the CFT [20] and accordingly identify an effective string tension and energy.

In Section 3, we consider a boundary theory of a rather different nature; the boundary now being the AdS-black hole horizon. Following Susskind's proposal of a black hole-string correspondence [21], we show that the near-horizon geometry can effectively be described by a single fundamental string that lives on the stretched horizon. In this process, the associated thermodynamics (including the string tension) are unambiguously identified.

In Section 4, we ultimately show that these two stringy descriptions of

the AdS-bulk are, indeed, thermodynamically equivalent. This entails a pair of coordinate rescalings with respect to the thermodynamic properties of the stretched horizon. The first rescaling is essentially a renormalization that accounts for the gravitational red shift occurring between the horizon and a cosmological observer. The second rescaling is necessary so that our hypothetical observer is located precisely at the CFT boundary.

Finally, Section 5 contains a summary and some commentary.

## 2 A CFT Description of AdS

Let us begin by considering the model of interest; namely, black hole solutions in an  $n+2$ -dimensional anti-de Sitter background (i.e.,  $D = n + 2$  Einstein gravity with a negative cosmological constant). In a suitably static gauge, such asymptotically AdS solutions can be described by the following line element [35]:

$$ds_{n+2}^2 = -h(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2d\Omega_n^2, \quad (1)$$

where:

$$h(r) = k + \frac{r^2}{L^2} - \frac{\omega_n M}{r^{n-1}}, \quad (2)$$

$$\omega_n = \frac{16\pi G}{n\mathcal{V}_n}. \quad (3)$$

Here,  $L$  is the curvature radius of the AdS background ( $L^{-2} = -2\Lambda/n(n+1)$ , with  $\Lambda$  as the negative cosmological constant),  $d\Omega_n^2$  denotes the line element of an  $n$ -dimensional hypersurface with a constant curvature  $kn(n-1)$  and a volume  $\mathcal{V}_n$ ,  $G$  is the  $n+2$ -dimensional Newton constant, and  $M$  and  $k$  are constants of integration.  $M$  is directly related to the ADM (or conserved) mass of the associated black hole, which we will assume is always non-negative.<sup>6</sup> Meanwhile,  $k$  describes the horizon geometry and, without loss of generality, can be set equal to +1, 0 or -1. These three choices correspond to a horizon geometry that is respectively spherical, flat or hyperbolic.

For a non-vanishing (positive)  $M$ , there will be a single black hole horizon, which corresponds to the positive root of  $h(r)$ . Denoting this horizon by

---

<sup>6</sup>Although, technically speaking, black hole solutions having negative mass are allowable in the case of a hyperbolic horizon geometry (i.e., the case of negative  $k$ ). [35, 36].

$r = R$ , we thus obtain the following useful relation:

$$k + \frac{R^2}{L^2} - \frac{\omega_n M}{R^{n-1}} = 0. \quad (4)$$

The associated black hole thermodynamics can readily be identified. For instance, if we disregard the energy of the  $M = 0$  vacuum spacetime, (which is non-vanishing if  $n$  is odd), then the excitation energy of the black hole is simply given as follows [37]:

$$E_{AdS} = M = \frac{1}{\omega_n} \left[ \frac{R^{n+1}}{L^2} + k R^{n-1} \right]. \quad (5)$$

Furthermore, the Hawking temperature [38] can be obtained with the usual prescription: the inverse temperature is equivalent to the periodicity of Euclidean time [39]. This identification yields:

$$T_{AdS} = \frac{1}{4\pi} \left. \frac{dh}{dr} \right|_{r=R} = \frac{R}{4\pi L^2} \left[ (n+1) + k(n-1) \frac{L^2}{R^2} \right]. \quad (6)$$

Finally, the black hole entropy can be expressed in terms of its “area” (i.e., the  $n$ -dimensional volume of the horizon hypersurface) via the Bekenstein-Hawking definition [1, 2]:

$$S_{AdS} = \frac{\mathcal{V}_n R^n}{4G} = \frac{4\pi}{n\omega_n} R^n. \quad (7)$$

Let us now reconsider the thermodynamics from a new perspective: an  $n+1$ -dimensional conformal field theory living on a timelike boundary of the bulk spacetime. In view of the well-established AdS/CFT correspondence [11, 12, 13], such a boundary theory can capably provide a holographic description of the asymptotically AdS geometry. In particular, the CFT thermodynamic quantities should be identifiable with those of the bulk black hole up to a simple red-shift factor. Because of the conformal symmetry of the dual theory, this red shift can vary depending on the choice of boundary coordinates. Here, we follow Verlinde [10, 15]<sup>7</sup> and fix the scale by setting the radial distance of the boundary equal to  $R$  (i.e., equal to the radius of

---

<sup>7</sup>Verlinde’s analysis was for  $k = +1$  only. The  $k \neq 0$  case was first considered by Cai [14].

the black hole horizon). This choice requires the bulk time to be rescaled (on the boundary) such that  $t \rightarrow tR/L$ , which directly translates into a corresponding red-shift factor of  $L/R$ .

On the basis of the above discussion, the CFT thermodynamics can be expressed as follows:

$$E_{CFT} = \frac{L}{R} E_{AdS} = \frac{1}{\omega_n} \left[ \frac{R^n}{L} + kLR^{n-2} \right], \quad (8)$$

$$T_{CFT} = \frac{L}{R} T_{AdS} = \frac{1}{4\pi L} \left[ (n+1) + k(n-1) \frac{L^2}{R^2} \right], \quad (9)$$

$$S_{CFT} = S_{AdS} = \frac{4\pi}{n\omega_n} R^n. \quad (10)$$

Note that the entropy of the boundary theory is never affected by the choice of scale [13], which complies with the first law of CFT thermodynamics.

Again following Verlinde [10], we will view the CFT energy as having arisen from a pair of separable contributions. More specifically,  $E_{CFT} = E_E + E_C$ , such that:

$$E_E = \frac{1}{\omega_n} \frac{R^n}{L}, \quad (11)$$

$$E_C = \frac{1}{\omega_n} kLR^{n-2}. \quad (12)$$

Of significance to this split,  $E_E$  can readily be identified as the extensive contribution to the total energy. Hence, the remaining portion,  $E_C$ , is the sub-extensive contribution, which is also known as the Casimir energy.<sup>8</sup>

With these definitions, it is not difficult to confirm a Cardy-like form for the entropy [4]; that is, the renowned Cardy-Verlinde formula [10] (slightly modified in allowing for  $k \neq +1$ ):

$$S_{CFT} = \frac{4\pi}{n} R \sqrt{\frac{E_C}{k}} [E_{CFT} - E_C]. \quad (13)$$

Note that  $E_C/k$  is to be regarded as a positive, non-vanishing quantity.

---

<sup>8</sup>Note that there is a factor of two difference between this definition of  $E_C$  and that of Verlinde's study [10].



It is also straightforward to verify the following form:

$$S_{CFT} = \frac{4\pi L}{n} E_E. \quad (14)$$

The significance of the above expression is the direct proportionality that exists between the CFT entropy and a well-defined energetic quantity,  $E_E$ . Such linearity is a characteristic behavior of strings [33], rather than black holes (for which entropy typically depends on mass as  $M^p$ , with  $p > 1$ ). With this in mind, we can re-interpret Eq.(14) as the equation of state for an “effective string” having an energy:

$$\mathcal{E}_{CFT} = \frac{2C}{n} E_E \quad (15)$$

and having a tension:

$$\mathcal{T}_{CFT} = \frac{C^2}{2\pi L^2}. \quad (16)$$

Here,  $C$  is a yet-to-be-determined parameter that is possibly a constant; however, we anticipate that, in general,  $C = C(R; n, k, L)$  [20].

The above string-like behavior is quite surprising, considering that the boundary theory has at least three spacetime dimensions (unless  $n = 1$ ). On the other hand, there has been considerable evidence that the relevant degrees of freedom for *any* black hole can be described by a two-dimensional conformal theory [5, 6].<sup>9</sup> It is possible that the above outcome is yet another manifestation of some unknown, universal principle that seems to be at work.

### 3 A Stringy Description of AdS

In the preceding section, we ultimately demonstrated that a dual CFT (with respect to an asymptotically AdS bulk) exhibits an intriguing string-like behavior. This observation segues nicely into our next topic; namely, a stringy description of asymptotically AdS spacetimes. Here, we will be applying the one-to-one correspondence principle, as first advocated by Susskind [21], between black holes and strings. The underlying premise is that a black hole and a fundamental string can respectively be regarded as strongly and weakly

---

<sup>9</sup>Also in support of this notion is the Cardy-Verlinde description itself (13), given that the original Cardy formula [4] has its genesis in two dimensions.

coupled versions of the same entity [23]. On this basis, Susskind has proposed that a massive black hole state can effectively be described by a highly excited string which (from the perspective of a cosmological observer) fills up the so-called stretched horizon [22]. Furthermore, Susskind has argued that the obvious differences (between black hole and string thermodynamics) can be attributed to a large gravitational red shift that occurs between the stretched horizon and an outside observer. (For subsequent work along these lines, see Refs.[25, 26, 27, 28, 29, 30, 31].)

The Susskind program [21] appears to apply quite naturally given any black hole (or black object for that matter) with a near-horizon geometry that can conform to a Rindler-like description. Let us now see how this plays out for a black hole in a AdS background of arbitrary dimensionality.

We begin here by reconsidering the asymptotically AdS geometry as described by Eqs.(1-3). To study the near-horizon form, let us introduce a new radial coordinate,  $y$ , in accordance with  $r = R + y$  (where  $y \ll R$  has been assumed). Up to first order in  $y/R$ , the metric function  $h(r)$  can now be written as:

$$h(y) \approx \frac{\Delta(R)}{L}y, \quad (17)$$

where we have defined:

$$\Delta(R) \equiv (n+1)\frac{R}{L} + k(n-1)\frac{L}{R}. \quad (18)$$

Applying the above, we find the following near-horizon form for the line element (1):

$$ds_{NH}^2 = -y\frac{\Delta(R)}{L}dt^2 + \frac{L}{y\Delta(R)}dy^2 + R^2d\Omega_n^2. \quad (19)$$

It is useful if  $y$  is replaced with a coordinate that directly measures the proper distance from any given point to the horizon. Denoting this proper distance as  $\rho$ , we have (up to the first perturbative order):

$$\rho \approx \sqrt{\frac{L}{\Delta}} \int^y \frac{dy}{\sqrt{y}} = 2\sqrt{\frac{Ly}{\Delta}}. \quad (20)$$

The near-horizon line element (19) now adopts the following Rindler-like form:

$$ds_{NH}^2 = -\frac{\Delta^2}{4L^2}\rho^2dt^2 + d\rho^2 + R^2d\Omega_n^2. \quad (21)$$

To obtain a near-horizon geometry that is identically Rindler spacetime:

$$ds_{NH}^2 = -\rho^2 dt^2 + d\rho^2 + R^2 d\Omega_n^2, \quad (22)$$

we simply redefine the time coordinate as follows:

$$\tau \equiv \frac{\Delta(R)}{2L} t. \quad (23)$$

Hence,  $\tau$  is the dimensionless Rindler time.

Next, we consider thermodynamics as measured by a hypothetical Rindler observer at the stretched horizon. The dimensionless horizon temperature is known to be [40]:

$$T_R = \frac{1}{2\pi}. \quad (24)$$

Meanwhile, the horizon entropy should still be given by the Bekenstein-Hawking area law (or its  $n+2$ -dimensional analogue) [1, 2]. Hence:

$$S_R = S_{AdS} = S_{CFT}. \quad (25)$$

To determine the dimensionless Rindler energy, we simply apply the first law of thermodynamics; that is,  $dE_R = T_R dS_R$ . This process yields:

$$E_R = \frac{1}{2\pi} S_R = \frac{2L}{n} E_E, \quad (26)$$

with the right-most relation following from Eqs.(14,25).

The linear relation between Rindler entropy and energy is (once again) indicative of a string; in this case, one that lives on the stretched horizon [21]. Note that the associated string tension, in dimensionless Rindler coordinates, is trivially given by:

$$\mathcal{T}_R = \frac{1}{2\pi}. \quad (27)$$

Of course, the dimensionless string energy is simply  $\mathcal{E}_R = E_R$ .

Clearly, the fundamental length scale for this effective theory is just the string length,  $l_s$  (as this determines the radial extent of the stretched horizon). Hence, we can obtain the thermodynamics of the stretched horizon by rescaling the dimensionless Rindler quantities in terms of this length. That

is, quantities with units of inverse length (or energy) should be divided by  $l_s$ . On this basis, the following identifications can readily be made:

$$T_{SH} = \frac{T_R}{l_s} = \frac{1}{2\pi l_s}, \quad (28)$$

$$S_{SH} = S_R = S_{AdS}, \quad (29)$$

$$\mathcal{E}_{SH} = \frac{\mathcal{E}_R}{l_s} = \frac{2L}{nl_s} E_E, \quad (30)$$

$$\mathcal{T}_{SH} = \frac{\mathcal{T}_R}{l_s^2} = \frac{1}{2\pi l_s^2}. \quad (31)$$

Note that, as expected,  $T_{SH}$  corresponds precisely with the Hagedorn temperature of a string [33].

We again point out that, according to Susskind [21], the discrepancy between the above thermodynamic quantities and those of the corresponding black hole (in this case, Schwarzschild-AdS or its topological variant) should be attributable to the effects of an immense gravitational red shift. This conjecture has, in fact, been rigorously demonstrated for a number of models (for instance, [26]) up to some ambiguity in the numerical coefficients. Alternatively, we will soon demonstrate the validity of this conjecture (in an AdS context) by comparing the thermodynamic properties for a pair of strings. These being the string living on the stretched horizon and the effective string located at the CFT boundary.

## 4 A String/CFT Correspondence?

So far, we have seen two different descriptions of an asymptotically AdS spacetime that are indicative of a string living on a boundary; with the boundaries in question being that which hosts a dual CFT and the stretched black hole horizon. The purpose of the current section is to demonstrate the equivalence of these effective theories, with thermodynamics serving as the testing ground. Keep in mind that there is no reason, *a priori*, to believe that these seemingly unrelated pictures should fundamentally coincide.

As an initial step, it is necessary that the thermodynamics of the stretched horizon be rescaled for an external or cosmological observer (who is presumed to be far away from the horizon in terms of all relevant length scales). The

simplest way to accomplish this task is to rescale the Rindler thermodynamic quantities (24-27) so that they are directly measured in terms of the “cosmological” time coordinate,  $t$ . Utilizing Eq.(23) (which directly relates  $t$  to the Rindler time,  $\tau$ ), we obtain the following rescaled quantities:

$$T'_{SH} = \frac{d\tau}{dt} T_R = \frac{\Delta(R)}{4\pi L}, \quad (32)$$

$$S'_{SH} = S_R = S_{AdS}, \quad (33)$$

$$\mathcal{E}'_{SH} = \frac{d\tau}{dt} \mathcal{E}_R = \frac{\Delta(R)}{n} E_E, \quad (34)$$

$$\mathcal{T}'_{SH} = \left( \frac{d\tau}{dt} \right)^2 \mathcal{T}_R = \frac{\Delta^2(R)}{8\pi L^2}. \quad (35)$$

Note that a prime indicates that a rescaling has taken place.

It is worthwhile for us to compare the above outcomes with those of Eqs.(28-31); that is, the thermodynamic properties of the stretched horizon as measured by a local observer. In going from the horizon perspective to the cosmological one, we observe an effective renormalization of roughly  $l_s^{-1} \rightarrow L^{-1}$ . Since it is usually assumed that  $L \gg l_s$ , this translates to a significant reduction in the energy, string tension and temperature as measured by a distant observer. Such a renormalization is, however, expected and can be attributed to a large gravitational red shift that naturally occurs between the horizon and an external vantage point [21].

Next, for the sake of comparison, we will consider an observer who is specifically located at the CFT boundary. Thus, to maintain consistency, it is necessary that the coordinates be rescaled so that the observer in question lives on a timelike hypersurface of fixed radial distance  $R$  (i.e., the radius of the black hole horizon). As previously discussed, this translates into a red-shift factor of  $L/R$  for any quantity having units of inverse length.

Let us begin here with the stretched-horizon entropy. As usual, this dimensionless quantity is unaffected by any coordinate rescalings, and so we trivially achieve the following agreement:

$$\begin{aligned} S''_{SH} = S'_{SH} &= S_{AdS} \\ &= S_{CFT}. \end{aligned} \quad (36)$$

Next, let us consider the temperature at the stretched horizon. The appropriate red shifting yields the following:

$$\begin{aligned} T_{SH}'' = \frac{L}{R} T_{SH}' &= \frac{\Delta(R)}{4\pi R} \\ &= \frac{1}{4\pi L} \left[ (n+1) + k(n-1) \frac{L^2}{R^2} \right]. \end{aligned} \quad (37)$$

Comparing with Eq.(9), we are able to make the anticipated identification:  $T_{SH}'' = T_{CFT}$ .

Also on the agenda is the rescaled value of the string tension. Here, we find:

$$\begin{aligned} \mathcal{T}_{SH}'' = \frac{L^2}{R^2} \mathcal{T}_{SH}' &= \frac{\Delta^2(R)}{8\pi R^2} \\ &= \frac{1}{8\pi L^2} \left[ (n+1) + k(n-1) \frac{L^2}{R^2} \right]^2. \end{aligned} \quad (38)$$

Let us assume that this rescaled string tension matches up with the CFT-inspired tension of Eq.(16). (The validity of this assumption will be tested via the energy.) This allows us to fix the parameter  $C = C(R)$ , and this process yields:

$$C(R) = \frac{1}{2} \left[ (n+1) + k(n-1) \frac{L^2}{R^2} \right] = \frac{L\Delta(R)}{2R}. \quad (39)$$

Finally, let us consider the rescaled value of the string energy. This calculation goes as follows:

$$\begin{aligned} \mathcal{E}_{SH}'' = \frac{L}{R} \mathcal{E}_{SH}' &= \frac{L\Delta(R)}{nR} E_E \\ &= \frac{2C(R)}{n} E_E, \end{aligned} \quad (40)$$

where we have incorporated the identification of  $C(R)$  into the lower line. Remarkably, this outcome is in complete agreement with the CFT-inspired string energy,  $\mathcal{E}_{CFT}$  of Eq.(15).

As an aside, it is interesting to consider the special case of  $n = 1$  (i.e., three-dimensional AdS spacetime), which (if  $k = 1$ ) describes a theory having

BTZ black hole solutions [41]. In this event,  $C$  takes on the simple value of 1 and, hence,  $\mathcal{E}_{SH}'' = \mathcal{E}_{CFT} = 2E_E$ .

Clearly, we have observed a precise correspondence between the renormalized thermodynamics of the stretched horizon and the thermodynamics of the conformal boundary theory. This apparent duality suggests that the string which lives on the CFT boundary (cf. Eqs.(15,16)) is really just the string at the stretched horizon as viewed by a boundary observer. This is a remarkable outcome, considering that we have utilized a pair of descriptions with **no** *a priori* relationship.

In spite of the success of this program, one might argue that the CFT “total” energy (i.e.,  $E_{CFT}$  of Eq.(8)), rather than the CFT-inspired string energy ( $\mathcal{E}_{CFT}$ ), should have been used to test the duality. If this is an accurate assessment, then there is an apparent failing in the correspondence, inasmuch as  $\mathcal{E}_{SH}'' = E_{CFT}$  can clearly **not** be satisfied (except for the very special choice of pure AdS with  $k = 1$ ). On the other hand, such a discrepancy seems very similar to the difference in energy when one compares calculations in, for instance, global and Poincare AdS coordinates [37]. That is to say, conserved charges in an AdS background are not so well defined (as they are in asymptotically flat spacetimes) and may indeed be subject to an ambiguous observer dependence.

Regardless of the viewpoint one takes on the above matter, it is still an interesting exercise to calculate the explicit difference between  $\mathcal{E}_{SH}''$  and  $E_{CFT}$ . First, let us apply Eqs.(39,40) and re-express the renormalized energy of the stretched horizon as follows:

$$\mathcal{E}_{SH}'' = \frac{R^n}{nL\omega_n} \left[ (n+1) + k(n-1) \frac{L^2}{R^2} \right]. \quad (41)$$

Also, let us recall Eq.(8) for the total energy of the CFT:

$$E_{CFT} = \frac{R^n}{nL\omega_n} \left[ n + kn \frac{L^2}{R^2} \right]. \quad (42)$$

Defining  $\delta E \equiv \mathcal{E}_{SH}'' - E_{CFT}$ , we then have:

$$\delta E = \frac{R^n}{nL\omega_n} \left[ 1 - k \frac{L^2}{R^2} \right] = \frac{1}{n} [E_E - E_C], \quad (43)$$

where Eqs.(11,12) have also been incorporated.

The simplicity of the above expression is quite intriguing. We suspect that there is some profound explanation as to why the “energy shift” takes on such a concise, universal form; however, the answer remains a mystery at the present time.

## 5 Conclusion

In summary, we have been studying an asymptotically anti-de Sitter space-time of arbitrary dimensionality. In particular, we have concentrated on static solutions having a single black hole horizon; that is, Schwarzschild-AdS black holes and their topological variants [35]. To begin the analysis, we introduced the relevant formalism and then derived the black hole thermodynamic expressions for this somewhat generic model. Following these preliminary considerations, we proceeded to investigate a pair of effective boundary theories that are related to the bulk theory via commonly accepted dualities. (We once again note that this work has been inspired by Haylo’s similar treatment in a de Sitter context [20].)

The first boundary theory under consideration was a conformal field theory that lives on a timelike hypersurface of the AdS bulk. Exploiting the AdS/CFT duality [11, 12, 13], we were able to identify the corresponding thermodynamic properties. In particular, we verified the celebrated Cardy-Verlinde formula [10] and, moreover, demonstrated a linear relation between the entropy and extensive energy of the CFT. Significantly, this linear relationship indicates that the CFT can effectively be viewed as a string that lives on this cosmological boundary. On this basis, we were able to identify an effective tension and energy for this stringy version of the conformal boundary theory.

The second boundary theory of interest was based on Susskind’s proposed correspondence [21] between any sufficiently massive black hole and a highly excited string that fills up the stretched horizon. This duality becomes evident when one transforms the near-horizon geometry of a black hole into a Rindler-like form. Following this program, we are able to calculate the thermodynamic properties of the (conjectured) string at the stretched horizon (which follow directly from the thermodynamics of the associated Rindler space). The black hole-string correspondence [21] then implies that a large gravitational red shift should account for the thermodynamic differences be-



tween this string state and the associated black hole.

The final portion of our analysis focused on comparing this pair of (seemingly unrelated) boundary theories. Such a comparison necessitated a two-step procedure. Firstly, we reconsidered the stretched-horizon thermodynamics from the perspective of a bulk observer (far away from the horizon). This required a coordinate transformation from Rindler time to “cosmological” time, for which the net effect was a significant renormalization of the dimensional thermodynamic properties. Such a renormalization (or red shifting) is expected by virtue of the long-range gravitational field that intervenes between the stretched horizon and the external observer. Secondly, for a direct comparison of these effective theories, it was necessary for our bulk observer to be located at the CFT-based hypersurface. This consideration was accounted for with one additional red shifting of the stretched-horizon thermodynamics.

Ultimately, we were able to show that the renormalized thermodynamics of the stretched horizon are indeed equivalent to the “stringy” CFT thermodynamics (provided that an arbitrary parameter has been suitably fixed). These identifications included the temperature, string energy, string tension and, trivially, the entropy. This remarkable equivalence implies that the thermodynamics on the CFT boundary can directly be attributed to a string that lives on the stretched horizon. This is a significant outcome given that the two boundary theories are *a priori* unrelated.

One issue of note is a finite shift that occurred between the (renormalized) string energy and the “standard energy” which is usually attributed to the CFT [10]. We have interpreted this discrepancy as being analogous to an ambiguous observer dependence that is inherent to asymptotically AdS spacetimes. That is, different AdS coordinate systems can often give rise to different definitions of energy (and other conserved charges) [37]. Interestingly, this energy shift took on a universal, simple form: essentially, the difference between the extensive and sub-extensive (or Casimir) contributions to the CFT energy. We expect that there is some profound explanation for this outcome and hope to return to this issue in a future study.

In conclusion, it is quite puzzling that a CFT of arbitrary dimensionality can be explained, at least effectively, by a single fundamental string; that is, a two-dimensional entity. On the other hand, this outcome nicely complements the success of the ( $n+1$ -dimensional) Cardy-Verlinde formula [10]; insofar as the original Cardy formula should, in principle, only apply to two-

dimensional CFTs [4]. The notion that a black hole can holographically be described by a two-dimensional CFT is certainly not a new one [3]. In fact, Carlip [5] and Solodukhin [6] have demonstrated that this is truly a universal feature of black hole geometries. The pertinent point is that, near a black hole horizon, the relevant physics appears to be restricted to the  $r$ - $t$  plane. It is remarkable that so many treatments, which are otherwise unrelated, can reach the same conclusion. (What is, perhaps, even more remarkable is the apparent encoding of this near-horizon information on a distant boundary. This, in a nutshell, is the power of the Holographic principle.) It seems most likely that these various viewpoints represent different descriptions of one fundamental theory. A better understanding of this phenomena would go a long way towards resolving the microscopic origin of black hole entropy.

## 6 Acknowledgments

The author would like to thank V.P. Frolov for helpful conversations.

## References

- [1] J.D. Bekenstein, Lett. Nuovo. Cim. **4**, 737 (1972); Phys. Rev. **D7**, 2333 (1973); Phys. Rev. **D9**, 3292 (1974).
- [2] S.W. Hawking, Comm. Math. Phys. **25**, 152 (1972); J.M. Bardeen, B. Carter and S.W. Hawking, Comm. Math. Phys. **31**, 161 (1973).
- [3] S. Carlip, Class. Quant. Grav. **15**, 3609 (1998) [hep-th/9806026]; “Black Hole Entropy from Horizon Conformal Field Theory”, gr-qc/9912118 (1999).
- [4] J.L. Cardy, Nucl. Phys. **B270**, 317 (1986).
- [5] S. Carlip, Phys. Rev. Lett. **82**, 2828 (1999) [hep-th/9812013]; Class. Quant. Grav. **16**, 3327 (1999) [gr-qc/9906126]; Phys. Lett. **B508**, 168 (2001) [gr-qc/0103100].
- [6] S.N. Solodukhin, Phys. Lett. **B454**, 213 (1999) [hep-th/9812056].

- [7] G. 't Hooft, “Dimensional Reduction in Quantum Gravity”, gr-qc/9310026 (1993).
- [8] L. Susskind, J. Math. Phys. **36**, 6377 (1995) [hep-th/9409089].
- [9] See, for instance, G. 't Hooft, Nucl. Phys. **B256**, 727 (1985).
- [10] E. Verlinde, “On the Holographic Principle in a Radiation Dominated Universe”, hep-th/0008140 (2000).
- [11] J.M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200].
- [12] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. **B428**, 105 (1998) [hep-th/9802109].
- [13] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150]; *ibid*, 505 (1998) [hep-th/9803131].
- [14] R.-G. Cai, Phys. Rev. **D63**, 124018 (2001) [hep-th/0102113].
- [15] I. Savonije and E. Verlinde, Phys. Lett. **B507**, 305 (2001) [hep-th/0102042].
- [16] S. Nojiri, S.D. Odintsov and S. Ogushi, “Cosmological and Black Hole Brane-World Universes in Higher-Derivative Gravity”, hep-th/0108172 (2001).
- [17] U.H. Danielsson, “A Black Hole Hologram in de Sitter Space”, hep-th/0110265 (2001).
- [18] A.J.M. Medved, “A Holographic Interpretation of Asymptotically de Sitter Spacetimes”, hep-th/0112226 (2001).
- [19] A. Strominger, JHEP **0110**, 034 (2001) [hep-th/0106113].
- [20] E. Haylo, “Strings and the Holographic Description of Asymptotically de Sitter Spaces”, hep-th/0201174 (2002).
- [21] L. Susskind, “Some Speculations about Black Hole Entropy in String Theory”, hep-th/9309145 (1993).

- [22] L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. **D48**, 3743 (1993) [hep-th/9306069].
- [23] G. 't Hooft, Nucl. Phys. **B335**, 138 (1990).
- [24] L. Susskind, “Strings, Black Holes and Lorentz Contraction”, hep-th/9308139 (1993).
- [25] E. Haylo, A.Rajaraman and L. Susskind, Phys. Lett. **B392**, 319 (1997) [hep-th/9605112].
- [26] E. Haylo, B. Kol, A.Rajaraman and L. Susskind, Phys. Lett. **B401**, 15 (1997) [hep-th/9609075].
- [27] G.T. Horowitz and J. Polchinski, Phys. Rev. **D55**, 6189 (1997) [hep-th/9612146]; Phys. Rev. **D57**, 2557 (1998) [hep-th/9707170].
- [28] E.Haylo, Int. J. Mod. Phys. **A14**, 3831 (1999) [hep-th/9610068]; Mod. Phys. Lett. **A13**, 1521 (1998) [hep-th/9611175]; “De Sitter Entropy and Strings”, hep-th/0107169 (2001); JHEP **0112**, 005 (2001) [hep-th/0108167].
- [29] T. Damour and G. Veneziano, Nucl. Phys. **B568**, 93 (2000) [hep-th/9907030].
- [30] T. Damour, “Quantum Strings and Black Holes”, gr-qc/0104080 (2001).
- [31] U. Danielsson, A. Guijosa and M. Kruczenski, JHEP **0109**, 011 (2001) [hep-th/0106201].
- [32] A. Strominger and C. Vafa, Phys. Lett. **B379**, 99 (1996) [hep-th/9601029].
- [33] A.W. Peet, “TASI Lectures on Black Holes in String Theory”, hep-th/0008241 (2000).
- [34] A. Sen, Mod. Phys. Lett. **A10**, 2081 (1995) [hep-th/9504147].
- [35] D. Birmingham, Class. Quant. Grav. **16**, 1197 (1999) [hep-th/9808032].
- [36] R. Emparan, JHEP **9906**. 036 (1999) [hep-th/9906040].

- [37] V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208**, 413 (1999) [hep-th/9902121].
- [38] S.W. Hawking, Comm. Math. Phys. **43**, 199 (1975).
- [39] G.W. Gibbons and S.W. Hawking, Phys. Rev. **D15**, 2752 (1977).
- [40] See, for instance, D. Bigatti and L. Susskind, “TASI Lectures on the Holographic Principle”, hep-th/0002044 (2000).
- [41] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992) [hep-th/9204099].